Iterative elimination of strictly dominated strategies

Perform iterative elimination of strictly dominated strategies (remember that you can use mixed strategies to eliminate) and find the Nash equilibria.

	A	**B**	**C**
X	2;0	3;10	4;4
Y	0;5	5;1	8;2
Z	1;1	2;3	0;1

Solution

For player 1, the pure strategy "Z" is strictly dominated by the pure strategy "X". The result yields the following matrix:

	A	**B**	**C**
X	2;0	3;10	4;4
Y	0;5	5;1	8;2

For player 2, if we combine pure strategies A and B by assigning a probability of 1/2 to each one, this new mixed strategy strictly dominates the pure strategy C.

$$0{,}5 * 0 + 0{,}5 * 10 > 4$$

$$0{,}5 * 5 + 0{,}5 * 1 > 2$$

	A	**B**
X	2;0	3;10
Y	0;5	5;1

There is no Nash equilibrium in pure strategies, so we move on to look for a Nash equilibrium in mixed strategies. Let p be the probability of player 1 playing X, and q the probability of player 2 playing A, we propose the following:

$$u_1(X;(q,1-q)) = 2*q + 3*(1-q) = 3-q$$

$$u_1(Y;(q,1-q)) = 0*q + 5*(1-q) = 5-5q$$

For player 1 to be indifferent between playing X or Y:

$$3-q = 5-5q$$

$$4q = 2$$

$$q = \frac{1}{2}$$

$$1-q = \frac{1}{2}$$

Now we are going to do the same exercise but to find the corresponding probabilities for player 1.

$$u_2((p,1-p);A) = 0p + 5(1-p) = 5-5p$$

$$u_2((p,1-p);B) = 10p + 1(1-p) = 1+9p$$

For player 2 to be indifferent between playing any of his strategies:

$$5-5p = 1+9p$$

$$4 = 14p$$

$$p = \frac{2}{7}$$

$$1-p = \frac{5}{7}$$

Therefore, the Nash equilibrium in mixed strategies is:

$$\left\{\left[(\frac{2}{7},\frac{5}{7});(\frac{1}{2},\frac{1}{2})\right]\right\}$$